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Various Results of the Diophantine Equations $x^2 - p = 2^n$ And $x^3 - p = 2^n$

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ABSTRACT:
Number Theory is that branch of Mathematics which deals with the properties of integers, more specifically the properties of positive integers. It has always occupied a unique position in the world of Mathematics. This is due to the historical importance of the subject. It is one of the few disciplines having demonstrable results. Since classical antiquity nearly every century has witnessed new and fascinating discoveries relating to the properties of numbers. Most of the great masters of Mathematical Sciences, at some point in their careers, have contributed to Number Theory. Number Theory held an irresistible appeal for the leading mathematicians due to the basic nature of its properties.

INTRODUCTION:

Famous mathematician S. Ramanujan conjectured that the Diophantine equation $x^2 + 7 = 2^n$ has only 5 positive integral solutions given by (x,n) where x is 1,3,5,11,181 and n is 3,4,5,7,15 respectively. Since x is odd the given equation can be written as $(2y-1)^2 = 2^n - 7$, with solutions 1,2,3,6 and 91 for y. This conjecture given by Ramanujan was proved by Nagell.

M. Mahanthapa, the high school student from Boulder Colorado considered the Diophantine equation $x^3 + 3 = 4^n$. He showed that this equation has only one positive integral solution given by x=1 and n=1.

Thien Do discussed a family of Diophantine equation in general form given by $x^2 + p = 2^n$, where p is an odd integer. He obtained the following results:

(i)For the equation $x^2 + p = 2^n$ to have positive integral solutions, x must be an odd integer and x^2 must be congruent to 1 modulo 8.

(ii)If p is an odd prime greater than 3 and not congruent 7(mod8) then the equation has no solution.

(iii)If p is 3 then the equation has a unique solution (1,2).

In this chapter, an attempt has been made to discuss the solution of the Diophantine equation $x^2 - p = 2^n$. Another form of Ramanujan Problem has also been obtained.

ANOTHER FORM OF RAMANUJAN PROBLEM:

The Diophantine equation $x^2 + 7 = 4^n$ has a unique solution given by x=3 and n=2.

Proof: The Ramanujan problem $x^2 + 7 = 2^n$ has a unique solution for an even value of n which is 4 and it can be written as 2^2. Thus if we consider the Diophantine equation $x^2 + 7 = 4^n$, it has unique solution given by x=3 and n=2.

Theorem 1:

If p is an odd prime then the Diophantine equations $x^2 - p = 2^n$ and $x^3 - p = 2^n$ have odd solutions.

Proof: The given Diophantine equations imply that $x^2 = p + 2^n$ and $x^3 = p + 2^n$. Obviously the right hand side of these Diophantine equations is odd numbers when *p* is odd prime. Since powers of odd integers are odd, *x* must be odd.

Solution of the Diophantine equation $x^2 - 3 = 2^n$**:**

This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 5 = 2^n$**:**

This Diophantine equation has the solution (3,2).

Solution of the Diophantine equation $x^2 - 7 = 2^n$**:**

Here *p*=7 is a prime number which is congruent to 7(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put *x*=3, *n*=1 then the given Diophantine equation is satisfied. Thus (*x*,*n*)=(3,1) is the solution of the equation $x^2 - 7 = 2^n$.

Solution of the Diophantine equation $x^2 - 11 = 2^n$**:**

This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 13 = 2^n$**:**

This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 17 = 2^n$**:**

Here *p*=17 is a prime number which is congruent to 1(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put *x*=5, *n*=3 and *x*=9, *n*=6 then the given Diophantine equation is satisfied. Thus (*x*,*n*)=(5,3) and (9,6) are the solutions of the equation $x^2 - 17 = 2^n$.

Solution of the Diophantine equation $x^2 - 19 = 2^n$**:**

This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 23 = 2^n$**:**

Here *p*=23 is a prime number which is congruent to 7(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put *x*=5, *n*=1 then the given Diophantine equation is satisfied. Thus (*x*,*n*)=(5,1) is the solutions of the equation $x^2 - 23 = 2^n$.

Solution of the Diophantine equation $x^2 - 29 = 2^n$**:**

This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 31 = 2^n$**:**

This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 37 = 2^n$**:**

This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 41 = 2^n$**:**

Here *p*=41 is a prime number which is congruent to 1(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put *x*=7, *n*=3 and *x*=13, *n*=7 then the given Diophantine equation is satisfied. Thus (*x*,*n*)=(7,3) and (13,7) are the solutions of the equation $x^2 - 17 = 2^n$.

Solution of the Diophantine equation $x^2 - 43 = 2^n$**:**

This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 47 = 2^n$**:**

Here *p*=47 is a prime number which is congruent to 7(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put *x*=7, *n*=1 then the given Diophantine equation is satisfied. Thus (*x*,*n*)=(7,1) is the solutions of the equation $x^2 - 47 = 2^n$.

Solution of the Diophantine equation $x^2 - 53 = 2^n$**:**

This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 59 = 2^n$**:**

This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 61 = 2^n$ **:**
This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 67 = 2^n$ **:**
This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 71 = 2^n$ **:**
This Diophantine equation has no solution.

Solution of the Diophantine equation $x^2 - 73 = 2^n$ **:**
Here p=73 is a prime number which is congruent to 1(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put x=9, n=3 then the given Diophantine equation is satisfied. Thus (x,n)=(9,3) is the solutions of the equation $x^2 - 73 = 2^n$**.**

Solution of the Diophantine equation $x^2 - 79 = 2^n$ **:**
Here p=79 is a prime number which is congruent to 7(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put x=9, n=1 then the given Diophantine equation is satisfied. Thus (x,n)=(9,1) is the solutions of the equation $x^2 - 79 = 2^n$**.**

Solution of the Diophantine equation $x^2 - 89 = 2^n$ **:**
Here p=89 is a prime number which is congruent to 1(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put x=11, n=5 then the given Diophantine equation is satisfied. Thus (x,n)=(11,5) is the solutions of the equation $x^2 - 89 = 2^n$**.**

Solution of the Diophantine equation $x^2 - 97 = 2^n$ **:**
Here p=97 is a prime number which is congruent to 1(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put x=15, n=7 then the given Diophantine equation is satisfied. Thus (x,n)=(15,7) is the solutions of the equation $x^2 - 97 = 2^n$**.**

Solution of the Diophantine equation $x^2 - 101 = 2^n$ **:**
This Diophantine equation has no integral solution.

Solution of the Diophantine equation $x^2 - 103 = 2^n$ **:**
Here p=103 is a prime number which is congruent to 7(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put x=, n= then the given Diophantine equation is satisfied. Thus (x,n)=() is the solutions of the equation $x^2 - 103 = 2^n$**.**

Solution of the Diophantine equation $x^2 - 107 = 2^n$ **:**
This Diophantine equation has no integral solution.

Solution of the Diophantine equation $x^2 - 109 = 2^n$ **:**
This Diophantine equation has no integral solution.

Solution of the Diophantine equation $x^2 - 113 = 2^n$ **:**
Here p=113 is a prime number which is congruent to 1(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put x=11, n=3 and x=25, n=9 then the given Diophantine equation is satisfied. Thus (x,n)=(11,3) and (x,n)=(25,9) are the solutions of the equation $x^2 - 113 = 2^n$**.**

Solution of the Diophantine equation $x^2 - 127 = 2^n$ **:**
Here p=127 is a prime number which is congruent to 7(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put x=, n= then the given Diophantine equation is satisfied. Thus (x,n)=() is the solutions of the equation $x^2 - 127 = 2^n$**.**

Solution of the Diophantine equation $x^2 - 151 = 2^n$ **:**
Here p=151 is a prime number which is congruent to 7(mod8). Therefore the positive integral solutions of the given equation may be possible.

Solution of the Diophantine equation $x^2 - 167 = 2^n$ **:**

Here p=167 is a prime number which is congruent to 7(mod8). Therefore the positive integral solutions of the given equation may be possible. If we put x=13, n=1 then the given Diophantine equation is satisfied. Thus (x,n)=(13,1) is the solutions of the equation $x^2 - 167 = 2^n$.

Solution of the Diophantine equation $x^2 - 191 = 2^n$ **:**

Here p=191 is a prime number which is congruent to 7(mod8). Therefore the positive integral solutions of the given equation may be possible.

Solution of the Diophantineequation $x^2 - 193 = 2^n$ **:**

Here p=193 is a prime number which is congruent to 1(mod8). Therefore the positive integral solutions of the given equation may be possible.

Solution of the Diophantine equation $x^2 - 199 = 2^n$ **:**

Here p=199 is a prime number which is congruent to 7(mod8). Therefore the positive integral solutions of the given equation may be possible.

Solution of the Diophantine equation $x^3 - p = 2^n$ **:**

Solution of the Diophantine equation $x^3 - 11 = 2^n$ **:**

This Diophantine equation has the solution (3,4).

Solution of the Diophantine equation $x^3 - 19 = 2^n$ **:**

This Diophantine equation has the solution (3,3).

Solution of the Diophantine equation $x^3 - 23 = 2^n$ **:**

This Diophantine equation has the solution (3,2).

Solution of the Diophantine equation $x^3 - 61 = 2^n$ **:**

This Diophantine equation has the solution (5,6).

Solution of the Diophantine equation $x^3 - 109 = 2^n$ **:**

This Diophantine equation has the solution (5,4).

Solution of the Diophantine equation $x^3 - 311 = 2^n$ **:**

This Diophantine equation has the solution (7,5).

Solution of the Diophantine equation $x^3 - 727 = 2^n$ **:**

This Diophantine equation has the solution (9,1).

Solution of the Diophantine equation $x^3 - 601 = 2^n$ **:**

This Diophantine equation has the solution (9,7).

CONCLUDING REMARKS:

In this paper, the Diophantine equation $x^2 - p = 2^n$ has been solved for p=5, 7, 17, 23, 41, 47, 73, 79, 89, 97, 103, 113, 127, 151, 167, 191, 193 and 199. The Diophantine equation $x^3 - p = 2^n$ are obtained for p=11, 19, 23, 61, 109, 311, 601 and 727. These Diophantine equations can further be solved for other values of p. Another form of Ramanujan Diophantine equation has also been obtained.

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